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DYNAMICS OF ELASTIC ELECTRICALLY CONDUCTING SHELLS IN CONSTANT AND NON-STATIONARY MAGNETIC FIELDS*

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A system of non-linear equations of the electromechanics of thin elastic shells of finite conductivity is obtained by the asymptotic integration of Maxwell equations (in the quasistationary approximation) and the equations of the theory of elasticity by using the relative half-thickness η as a small parameter. It is shown how two of their fundamental linear limit forms corresponding to two known classes of problems: 1) determining the influence of a permanent magnetic field on the free vibrations of elastic shells /1/, and 2) the determination of the shell deformation due to ponderomotive forces caused by eddy currents indiced by alternating magnetic fields /2-4/, can be obtained from these equations by neglecting asymptotically small terms. A system of boundary conditions is given, and initial conditions for certain of the problems 2. Deductions are made from an analysis of the asymptotic accuracy about the limits of applicability of the equations obtained (and also of analogous linear equations obtained by different authors /1-3/). It is shown that the accuracy of any linear eqµations corresponding to problems 1 or 2 cannot be greater than $O(\eta)$.

1. Formulation of the problem.

A triorthogonal coordinate system $(\alpha_1, \alpha_2, \alpha_3)$ is given in an unbounded space V such that its coordinate surface $\alpha_3 = 0$ agrees with the middle surface S of a certain shell /5/. This shell occupies the domain $V^{(i)}$ bounded by facial surfaces given by the equalities $\alpha_3 = \pm h$ and closed by an edge surface defined by the equation $\varphi(\alpha_1, \alpha_2) = 0$. The external domain $V^{(e)} = V - V^{(i)}$ is occupied by a substance whose properties are identified with the properties of a vacuum, while the internal domain $V^{(i)}$ is filled by a material with linear elastic properties, finite electrical conductivity σ, and relative magnetic permeability of one.

We examine the problem of determining the vibrations of a small elastic shell in a given variable magnetic field.

Neglecting displacement currents and assuming there are no secondary currents, we write Maxwell's equations in the form /6/

$$-\Delta \mathbf{B}_\Sigma + \mu_0\sigma \mathbf{B}_\Sigma^{\boldsymbol{\cdot}} = \mu_0\sigma \operatorname{rot}(\mathbf{u}^{\boldsymbol{\cdot}} \times \mathbf{B}_\Sigma), \quad \operatorname{div} \mathbf{B}_\Sigma = 0 \tag{1.1}$$

Here $\mathbf{B}_\Sigma$ is the total magnetic induction vector, $\mathbf{u}$ is the elastic displacements vector of the medium, and μ_0 is a magnetic constant; the dot denotes a derivative with respect to time t.

We shall assume that

$$\mathbf{B}_\Sigma = \mathbf{B} + \mathbf{b} \tag{1.2}$$

where $\mathbf{B} = \mathbf{B}(\alpha_1, \alpha_2, \alpha_3, t)$ is generally the induction of the alternating magnetic field given in the whole domain V by solving the electrodynamic problem for a certain secondary field source when there is no shell (i.e., assuming that the whole space V has the properties of a vacuum), $\mathbf{b}$ is the induction of the desired current field in the shell (we later understand $\mathbf{b}^{(e)}$ and $\mathbf{b}^{(i)}$ to be the values of $\mathbf{b}$ in the domains $V^{(e)}$ and $V^{(i)}$, respectively).

In particular, this means that $\mathbf{B}$ everywhere satisfies the equations

$$\Delta \mathbf{B} = 0, \ \operatorname{div} \mathbf{B} = 0, \ \operatorname{rot} \mathbf{B} = 0 \tag{1.3}$$

When this is taken into account, after substituting (1.2) Eqs.(1.1) can be written in the form

$$\Delta \mathbf{b}^{(e)} = 0, \quad \operatorname{div} \mathbf{b}^{(e)} = 0 \quad \text{in} \quad V^{(e)} \tag{1.4}$$

$$-\Delta \mathbf{b}^{(i)} + \mu_0\sigma \mathbf{b}^{\boldsymbol{\cdot}(i)} = \mu_0\sigma\{-\mathbf{B}^{\boldsymbol{\cdot}} + \operatorname{rot}[\mathbf{u}^{\boldsymbol{\cdot}} \times (\mathbf{B} + \mathbf{b}^{(i)})]\} \tag{1.5}$$
$$\operatorname{div} \mathbf{b}^{(i)} = 0 \quad \text{in} \quad V^{(i)}$$

We will write the equations the dynamics of an elastic medium occupying the domain $V^{(i)}$

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in the form

$$(\lambda + \mu)\operatorname{grad}\operatorname{div}\mathbf{u} + \mu\Delta\mathbf{u} - \rho\mathbf{u}^{\cdot\cdot} = -\mathbf{q} \tag{1.6}$$

where λ and μ are the Lamé elastic constants, ρ is the density of the shell material, and $\mathbf{q}$ is the volume ponderomotive force vector expressed in terms of the current density j. Taking (1.2) and (1.3) into account, the formulas for finding them can be written in the form

$$\mathbf{q} = \mathbf{j} \times (\mathbf{B} + \mathbf{b}^{(i)}), \quad \mathbf{j} = \mu_0^{-1}\operatorname{rot}\mathbf{b}^{(i)} \tag{1.7}$$

Eqs. (1.4)-(1.6) must be integrated over the whole domain V when certain conditions are satisfied on the facial and edge surfaces. Some of these conditions are of an electromagnetic nature and can be reduced, in the case under consideration, to the equations

$$\mathbf{b}^{(e)} = \mathbf{b}^{(i)}, \quad \mathbf{n}\cdot\operatorname{rot}\mathbf{b}^{(i)} = 0 \tag{1.8}$$

where $\mathbf{n}$ is the vector of the normal to the surface separating the domains $V^{(i)}$ and $V^{(e)}$ (if the shell edge surface is not insulated, then the electromagnetic condition thereon will differ from (1.8)). Part of the conditions is mechanical in nature /5/ and is imposed on certain displacement functions.

The condition of boundedness at infinity is additionally imposed on the induction $\mathbf{b}^{(e)}$.

By solving the problem in question, all the remaining parameters of the electromagnetic and mechanical fields can be determined by direct operations using known formulas /5, 6/ of the corresponding theories.

This formulation of the problem differs from that presented in /1/ solely in the fact that the induction $\mathbf{B}$ can be variable, and the equations are not linearized in the domain $V^{(i)}$ by neglecting $\mathbf{b}^{(i)}$ compared with $\mathbf{B}$.

2. Asymptotic integration. We will consider the equations of Sect.1 by transferring to the independent variables $\xi_k, \zeta, \tau_s, \tau_e$ therein by means of the formulas

$$\begin{gathered}\alpha_k = \eta^p R\xi_k, \quad \alpha_3 = \eta R\zeta \\ t = \eta^l\mu_0\sigma R^2\tau_e \ (\text{в } (1.5)), \quad t = \eta^r\sqrt{\rho/E}\,R\tau_s \ (\mathbf{b}\ (1.6))\end{gathered} \tag{2.1}$$

where $\eta = h/R$ is the relative half-thickness, R is the characteristic radius of the shell curvature, and p, l, r are numbers that will be determined below. The subscripts k, m, i, j used here and henceforth take the values $k,\ m = 1, 2, k \neq m, i, j = 1, 2, 3$.

We refer the surface S to the lines of curvature. Then the Lamé coefficients are given by the formulas

$$H_k = A_k a_k, \quad a_k = 1 + \alpha_3/R_k, \quad H_3 = 1 \tag{2.2}$$

where R_k are the normal radii of curvature of the surface S. As in /5/, the different combinations of a_1, a_2 in the equations, which are obtained by multiplication or division, will be replaced by series of the form

$$f(a_1, a_2) = \sum (f)_n(\eta\zeta)^n, \quad (f)_n = (n!)^{-1}[\partial^n f/\partial(\eta\zeta)^n]_{\zeta=0} \tag{2.3}$$

Here and henceforth Σ always denotes summation between $n = 0$ and $n = \infty$.

When determining the terms of the expansion (2.3), α_3 in (2.2) must be replaced as given by formulas (2.1). We then obtain, in particular

$$\begin{gathered}(f)_0 = 1, \quad (a_1a_2)_1 = \frac{R}{R_1} + \frac{R}{R_2}, \quad (a_k)_1 = -\left(\frac{1}{a_k}\right)_1 = \frac{R}{R_k}, \\ (a_1a_2)_2 = \frac{R^2}{R_1R_2}\end{gathered}$$

We will represent the components of given vector $\mathbf{B}$ in the form of the expansions

$$\begin{gathered}B_j = B\sum(\eta\zeta)^n B_{jn}, \quad B_{jn} = B^{-1}[\partial^n B_j/\partial\alpha_3^{\,n}]_{\alpha_3=0} \\ B = \max_S|\mathbf{B}|\end{gathered} \tag{2.4}$$

and we will seek the components of the vectors $\mathbf{b}^{(i)}$ and $\mathbf{u}$ in the form

$$\begin{gathered}b_j^{(i)} = B\eta^c\sum\zeta^n\eta^{c_{jn}}b_{jn}, \quad u_j = R\sum\zeta^n\eta^{d_{jn}}u_{jn} \\ b_{jn} = b_{jn}(\alpha_1, \alpha_2, t), \quad u_{jn} = u_{jn}(\alpha_1, \alpha_2, t)\end{gathered} \tag{2.5}$$

where c, c_{jn}, d_{jn} are numbers whose values are determined below.

We substitute (2.1)-(2.5) into (1.5) and we satisfy these equations by satisfying the equations obtained by successive matching of the coefficients of identical powers of ζ therein (we call them the ζ-equations). We shall here consider the numbers p, l, r to have been selected so that differentiation of the desired functions with respect to ξ_k, τ_e, τ_s will not result in a change in their asymptotic orders. (In particular, this means that p agrees in meaning with the index of variability /5/ of the desired state). We take into account that according to (2.3) and (2.4) the quantities $(f)_n$ and B_{jn} are of the order of η^0. Moreover, we assume that the numbers c, c_{jn}, d_{jn} are selected such that the quantities b_{jn} and u_{jn} determined from the ζ-equations are also of the order of η^0. Then all the coefficients of these functions in the ζ-equations will have the structure $\eta^\varkappa P$, where P are certain operators or multipliers that do not influence the asymptotic form of the corresponding term, while $\eta^\varkappa$ is a multiplier governing its asymptotic order. Only the asymptotically principal components (containing η to the lowest power) allowing a certain estimatable error here, can be contained in each of the ζ-equations. The equations obtained in this manner should be consistent, i.e., several different expressions should not be obtained to determine the very same quantity, and the quantities determined themselves from these equations should be of the order of η^0.

The equations obtained from (1.5) by this method can satisfy the last condition if

$$c_{30} = c_{k0} = c_{k1} = c_{k2} = 0, \quad c_{31} = 1, \quad c_{32} = 1 - p, \quad d_{j0} = 1 \tag{2.6}$$

is taken for the first terms of the expansions (2.5).

From the ζ-equations obtained by equating the coefficients of ζ^0 and ζ in the second and of ζ^0 in the first of Eqs.(1.5), we obtain the following relationships after some identity manipulations:

$$\begin{aligned} b_{31} &= -[(a_1a_2)_1 b_{30} + \eta^{-p}(P_{01}b_{10} + P_{02}b_{20})] \\ b_{32} &= -\{(P_{01}b_{11} + P_{02}b_{21}) + \eta^{1+p}[(a_1a_2)_1 b_{31} + (a_1a_2)_2 b_{30}] + \\ &\quad \eta(P_{11}b_{10} + P_{12}b_{20})\}/2 \end{aligned} \tag{2.7}$$

$$\begin{aligned} &b_{k2} = [-\eta^{2-c+g}R^2Q_{k0} - \eta(a_1a_2)_1 b_{k1} - \eta^{2-2p}\Delta_\xi b_{k0} + \eta^{2-l}\partial b_{k0}/\partial\tau_e]/2 \\ &\eta^{c-1-p}(P_{01}b_{11} + P_{02}b_{21}) + \eta^{c-p}(P_{11}b_{10} + P_{12}b_{20}) + \\ &\quad \eta^c[\eta^{-l}\partial/\partial\tau_e - \eta^{-2p}\Delta_\xi - (a_1a_2)_2]\, b_{30} = \eta^g R^2 Q_{30} \end{aligned}$$

Here

$$\begin{aligned} &\Delta_\xi b_{j0} = \frac{1}{A_1A_2}\left[\frac{\partial}{\partial\xi_1}\left(\frac{A_2}{A_1}\frac{\partial b_{j0}}{\partial\xi_1}\right) + \frac{\partial}{\partial\xi_2}\left(\frac{A_1}{A_2}\frac{\partial b_{j0}}{\partial\xi_2}\right)\right] \\ &P_{0k}b_{jn} = \frac{1}{A_1A_2}\frac{\partial}{\partial\xi_k}(A_m b_{jn}), \quad P_{1k}b_{jn} = \\ &\quad \frac{1}{A_1A_2}\left[\left(\frac{1}{a_k}\right)_1 \frac{\partial}{\partial\xi_k} + \eta^p (a_m)_1\right](A_m b_{jn}) \end{aligned}$$

and Q_{j0} are the first terms of the expansions $Q_j = B\eta^g \Sigma\, \zeta^n \eta^{g_{jn}} Q_{jn}$ of the right-hand sides of the first equation in (1.5), obtained by substituting the relationships (2.1) and the expansions (2.3)-(2.5) therein. In particular (taking (2.6) into account), we have

$$\begin{aligned} \eta^g Q_{30} = \frac{1}{R^2}\Big\{&-\eta^{-l}\frac{\partial B_{30}}{\partial\tau_e} + \eta^{1-l-p}\frac{1}{A_1A_2}\Big\{\frac{\partial}{\partial\xi_1}\Big[A_2\Big((B_{10} + \eta^c b_{10}) \times \\ &\frac{\partial u_{30}}{\partial\tau_e} - (B_{30} + \eta^c b_{30})\frac{\partial u_{10}}{\partial\tau_e}\Big)\Big] + \frac{\partial}{\partial\xi_2}\Big[A_1\Big((B_{20} + \eta^c b_{20}) \times \\ &\frac{\partial u_{30}}{\partial\tau_e} - (B_{30} + \eta^c b_{30})\frac{\partial u_{20}}{\partial\tau_e}\Big)\Big]\Big\}\Big\} \end{aligned} \tag{2.8}$$

The last equation in (2.7) connects the terms of the expansion (2.5) for the induction of the desired current field to the left of the equality sign with the components governing the perturbations causing these currents that are on the right-hand side. Hence, the number c giving the asymptotic form (2.5) of the induction $\mathbf{b}^{(i)}$ is selected so that at least one of the components on different sides of the equality sign will be among the number of asymptotically principal terms in the last equation in (2.7). This condition is satisfied by

$$c = \begin{cases} 1 + p + g, & l < 1 + p \\ l + g, & 1 + p \leqslant l \end{cases} \tag{2.9}$$

Using (2.9) we can determine the relationship between the asymptotic orders of any pair of components in each of Eqs.(2.7). We will neglect terms

$$O(\eta^{\varepsilon_e}), \quad \varepsilon_e = \min(2 - l, 1 - p) \tag{2.10}$$

compared with the asymptotically principal terms in the equations below, i.e., we will construct equations to the accuracy of the quantities (2.10). When comparing the exponents in (2.7) we shall consider p and l bounded by the inequalities $0 \leqslant p < 1$ and $l < 2$, whose meaning will be given below.

Let us consider the problem of satisfying the electromagnetic conditions on linear surfaces.

We satisfy (1.4) by assuming that

$$\mathbf{b}^{(e)} = \operatorname{grad}\Phi = \frac{1}{H_1}\frac{\partial\Phi}{\partial\alpha_1}\mathbf{i}_1 + \frac{1}{H_2}\frac{\partial\Phi}{\partial\alpha_2}\mathbf{i}_2 + \frac{\partial\Phi}{\partial\alpha_3}\mathbf{i}_3 \tag{2.11}$$

where $\mathbf{i}_j$ are the unit directions along α_j, and Φ is a potential function satisfying the Laplace equation in the domain $V^{(e)}$.

Satisfying the first condition of (1.8) to an accuracy corresponding to (2.10), and taking acount of (2.3), (2.5), (2.6), (2.7) and (2.11), we obtain the expressions

$$B2\eta^c b_{k1} = \frac{1}{A_k}\frac{\partial}{\partial\alpha_k}(\Phi^+ - \Phi^-), \quad B\eta^c b_{30} = \left(\frac{\partial\Phi}{\partial\alpha_3}\right)^+ = \left(\frac{\partial\Phi}{\partial\alpha_3}\right)^- \tag{2.12}$$
$$B2\eta^c b_{k0} = \frac{1}{A_k}\frac{\partial}{\partial\alpha_k}(\Phi^+ + \Phi^-) \quad (\;)^\pm = (\;)_{\alpha_3\to\pm 0}$$

Because of expressions (2.12) obtained for b_{k0} and b_{k1}, it can be shown that the second condition of (1.8) is satisfied to an accuracy exceeding (2.10) by substituting (2.5) therein.

Discarding small terms when substituting (2.11) into the first equation of (1.8) means that the domain $V^{(i)}$ is reduced to a mathematical slit in S. The error here equals $O(\eta)$, i.e., it corresponds to (2.10).

We return to (1.6). Integration of its corresponding system of scalar equations in the slender domain $V^{(i)}$ (occupied by the shall material) under the appropriate conditions on the facial surfaces is the ordinary problem of constructing the equations of dynamic shell theory and is realized by stretching the scale using formulas (2.1). This problem can be formulated in the terminology of /5/, say, if the volume forces are understood to be ponderomotive forces together with inertial forces. The equations obtained here can possess an accuracy no lower than

$$O(\eta^{\varepsilon_s}), \; \varepsilon_s = \min(2-2r,\, 1-p) \tag{2.13}$$

if the inertial terms are taken in the usual form /7/ and a magnetic pressure $\mathbf{X}$ is introduced with components determined from the formulas

$$X_j = 2hq_{j0} \tag{2.14}$$

where q_{j0} are the first terms of the expansion $q_j = \Sigma\zeta^n q_{jn}$ obtained by substituting (2.1)-(2.6) into (1.7). They have the form

$$q_{k0} = B^2(\mu_0 R)^{-1}\eta^{c-1}(B_{30} + \eta^c b_{30})\, b_{k1} \tag{2.15}$$
$$q_{30} = -B^2(\mu_0 R)^{-1}\eta^{c-1}[(B_{10} + \eta^c b_{10})\, b_{11} + (B_{20} + \eta^c b_{20})\, b_{21}]$$

We will write these equations by appending (2.12) and the last equation in (2.7) to them. (In passing, we omit the second component with the factor η^{c-p} in the last equation in (2.7); it is a quantity of the order of η compared with the first term containing the factor η^{c-1-p}). We make the reverse substitutions (2.1) in the equations and substitute $\eta^c b_{30}, \eta^c b_{k0}, \eta^c b_{k1}$ according to (2.12). Going back to dimensional quantities according to (2.4) and (2.5), we obtain the system of equations

$$\frac{1}{2h\mu_0\varsigma}\Delta_s F + f_3^{\cdot} = -j_2 B_3^{\cdot} + j_3\frac{1}{A_1A_2}\frac{\partial}{\partial\alpha_k}\{A_m[(B_k + j_1 f_k)\, u_3^{\cdot} - \tag{2.16}$$
$$(B_3 + j_1 f_3)\, u_k^{\cdot}]\}$$
$$\Delta\Phi = 0, \quad F = \Phi^+ - \Phi^-, \quad f_3 = \left(\frac{\partial\Phi}{\partial\alpha_3}\right)^+ = \left(\frac{\partial\Phi}{\partial\alpha_3}\right)^-$$

$$\left(\frac{h^2}{3}N_{ij} + L_{ij}\right)u_j + \frac{\rho}{E}u_i^{\cdot\cdot} = \frac{X_i}{2Eh} \tag{2.17}$$

$$X_k = \frac{1}{\mu_0}(B_3 + j_1 f_3)\frac{1}{A_k}\frac{\partial F}{\partial\alpha_k}, \quad X_3 = -\frac{1}{\mu_0}(B_k + j_1 f_k)\frac{1}{A_k}\frac{\partial F}{\partial\alpha_k} \tag{2.18}$$
$$f_k = \frac{1}{2A_k}\frac{\partial}{\partial\alpha_k}(\Phi^+ + \Phi^-)$$

Here L_{ij} and N_{ij} are the membrane and couple-stress operators of shell theory (they can be taken in the form given in /7/), Δ_s is the two-dimensional Laplace operator in S (it can be obtained from Δ_ξ by the formal replacement of ξ_k by α_k), B_j is understood to be the values of the components of the vector $\mathbf{B}$ on the surface S, u_j are displacements of the shell middle surface, the subscripts i, j, k, m are ascribed the values taken here and it is considered that summation is over repeated indices, while $j_1 - j_3$ are factors introduced for convenience in the exposition (although they must be considered to be equal to one).

To the accuracy of (2.10) the linear current $\mathbf{J}$ in a shell can be expressed in terms of the quantities (2.16)-(2.18) by means of the formula

$$\mathbf{J} = \frac{1}{\mu_0}\left[-\frac{1}{A_2}\frac{\partial F}{\partial \alpha_2}\mathbf{i}_1 + \frac{1}{A_1}\frac{\partial F}{\partial \alpha_1}\mathbf{i}_2\right] \tag{2.19}$$

Remarks. 1°. After (2.16)-(2.18) have been solved, when the function Φ has become known, all the terms of the expansions $b_j^{(i)}$ considered in Sect.2 can be determined by direct operations: b_{30}, b_{k0} and b_{k1} from (2.12), and b_{31}, b_{32}, b_{k2} from (2.7). To determine the next terms in the expansions (2.5) it is necessary to construct the next corresponding to (1.5) after the ζ-equations considered.

2°. The conditions of formal asymptotic convergence of the process in question is the requirement that the indices ε_e and ε_s of degree η in (2.10) and (2.13), which govern the asymptotic form of the discarded terms, should be positive. According to (2.1) this yields the following symbolic inequalities

$$\frac{\partial}{\partial \alpha_j} \ll \eta^{-1}, \ \frac{\partial}{\partial t} \ll \min(\omega_e, \omega_s), \quad \omega_e = (4h^2\mu_0\sigma)^{-1}, \quad \omega_s = (2h)^{-1}\sqrt{\frac{E}{\rho}}$$

constraining the properties of the processes under investigation to limits within which the two-dimensional dynamic theory of shells is valid and there is no skin effect.

3°. The boundary conditions for (2.16)-(2.18) can provisionally be separated into the following groups.

Mechanical conditions which can be understood to be the usual boundary conditions of shell theory expressed in terms of displacements of the middle surface. These conditions can be satisfied because of the arbitrariness of (2.17).

Electrodynamic conditions, one of which is the natural requirement of the boundedness of Φ at infinity, while the other is imposed on the value of certain quantities in (2.16)-(2.18) at the shell edge. In particular, where there should be no current along the normal to the edge on an insulated edge, this condition is satisfied, according to (2.19) if the edge value of the function F equals zero.

3. Limiting forms of the magnetoelasticity equations.

Eqs.(2.16)-(2.18) are a closed system of the equations of the dynamics of elastic thin-walled shells in magnetic fields and are non-linear because of the components containing the product of f_j and certain derivatives of F and u_j. We will show that (2.16)-(2.18) can be linearized for the solutions of problems *1* and *2* defined at the beginning of the paper (because of the neglect certain components) by thereby going over to certain limit forms.

The number g determining the asymptotic form of the sums Q_{j0} and, particularly, Q_{30} remains indefinite in the derivation of (2.16)-(2.18). Equating the index g of degree η in (2.8) on the left-hand side of the equations to the least of the indices of powers of the factor η on the right-hand side and taking into account that the first component on the right-hand side is identically zero in the problems *1* (the quantity B_{30} is constant with time), we obtain the formulas $g = 1 - p - l$ (in problems *1*) and $g = -l$ (in problems *2*). It follows from (2.9) that the minimum values of c governing the maximum asymptotic form of $b_j^{(i)}$ according to (2.5) are

$$c = 1 - p \text{ (in problems } 1), \ c = 0 \text{ (in problems } 2) \tag{3.1}$$

This means that for problems *1* the components $\eta^c b_{j0}$ can be neglected with accuracy $O(\eta^{1-p})$ as compared with B_{j0} in (2.8) and (2.15). We take into account also that in problems *1* $\partial B_{30}/\partial \tau_e = 0$. Neglecting the appropriate terms in (2.16)-(2.18), we obtain the limit form of the equations that corresponds to problems *1*. It follows from (2.16)-(2.18), if we set $j_1 = j_2 = 0, j_3 = 1$. (The functions f_k do not enter the equations here).

Discarding the components $\eta^c b_{j0}$ compared with B_{j0} in problems *2* is not legitimate since they can be of the identical asymptotic order. However, (to the accuracy of $O(\eta^{1-p})$) it is allowable to omit the second group of components on the right-hand side of (2.8). This means that the limit form of the equations corresponding to problems *2* follows from (2.16)-(2.18) if we set $j_3 = 0, j_1 = j_2 = 1$.

We note that according to (3.1), (2.5), (1.7) and (2.15), the eddy current density and magnetic pressure in problems *2* is η^{1-p} times greater than in problems *1* for an identical magnetic field intensity level.

4. Discussion of magnetoelasticity problems.

The limiting system of equations of problems *1* is coupled. It can be obtained from the equations given in /1/, if the tangential electrical field components in the shell are eliminated in them and asymptotically small terms are neglected /8/. Depending on the frequency of vibration and the magnetic field strength, these equations can be simplified additionally in the same way as in /9, 10/ with the magnetoelasticity equations of a three-dimensional body.

The limit system of equations for problems *2* uncouples and can always be solved in three successive steps. The first step is to determine the functions Φ, F, f_3 on the basis of

(2.16) ($j_3 = 0$). The second is to determine the magnetic pressure vector components X_j by means of (2.18) by direct operations. And the thrid is to integrate (2.17) for given right-hand sides and to find the shell displacements u_j. If necessary, the eddy currents can be found by means of (2.19) by direct operations after the first step.

This scheme is common for the following problems *2*.

1°. Determination of the steady vibrations in harmonically varying magnetic fields at the frequencies $\omega \ll \omega_{\max} = \min(\omega_e, \omega_s)$. Problems on vibrations in a field of variable currents, vibrations caused by the relative rotation of the field and the shell as well as their combinations are belong here. The expressions obtained for the displacements in these problems show the presence of resonances at twice the frequency of the electromagnetic excitation and, as a rule, contain a constant (time independent) component.

2°. Determination of the action of a smooth electromagnetic pulse /4/ satisfying the condition $\partial/\partial t \ll \omega_{\max}$, or equivalently, the duration $\tau \gg \omega_{\max}^{-1}$, on a shell. (For a steel shell $2h = 1$ mm thick $\tau \gg 2{,}2$ μsec). In this case the equations of the problem must be supplemented by homogeneous initial conditions on Φ, u_j and $u_j^{\cdot}$.

3°. The effect of connecting or disconnecting a permanent magnetic field on a shell. (The change in field can be considered as stepwise if $\partial/\partial t > \omega_e$ in the front). For these problems it is necessary to set $\mathbf{B}$ =const in these equations. The right-hand side ($B_3^{\cdot} = 0$ or formally $j_2 = 0$) vanishes in the first equation of (2.16) here. They must be supplemented by initial conditions which have the respective forms $f_3 = -B_3$ and $f_3 = B_3$ (for $t = 0$) for a step switch-on and switch-off of the magnetic field $\mathbf{B}$. The solution of this problem is the sum of exponentially decaying proper solutions of (2.16) in time ($j_2 = j_3 = 0$) and by means of (2.18) determines the exponentially decaying right-hand sides of (2.17). The initial conditions for u_j and $u_j^{\cdot}$ can be satisfied because of the arbitrariness of (2.17).

The fundamental mathematical difficulties associated with solving problems *2* are the integration of the three-dimensional Eqs.(2.16) which are one form of the eddy-current equations. Other forms obtained on the basis of hypotheses of eddy-current equations are contained in /2-4/. Thus, in substance, the equations in /2, 3/ are an integral form of Eqs.(2.16) corresponding to problem *2*. A literature survey on this questions is contained in /2/, and a method for the numerical solution of problems to determine the eddy currents in shells is also given. Analytic solutions are presented in /3/ for plates and shells of simple shape that are in harmonic and rotating fields. In certain cases formulas are given to determine the magnetic pressure.

The results obtained in each of the steps in solving problems *2* may be of independent practical value.

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